

Philosophy 211 -- Assignment #4

I. Prove these sequents. You may use any rules mentioned in the book including derived rules such as DeMorgan's Laws and $\text{Neg} \rightarrow$ which will be particularly helpful.

1. $P \rightarrow (Q \rightarrow R) \vdash \sim R \rightarrow \sim (P \& Q)$
2. $P \rightarrow (U \& S), Q \rightarrow (T \& \sim S) \vdash Q \rightarrow \sim P$
3. $\sim P \rightarrow Q, P \rightarrow Q, Q \rightarrow P \vdash P \& Q$
4. $P \vee Q \vdash (P \rightarrow Q) \rightarrow Q$
5. $P \rightarrow Q, \sim P \rightarrow R \vdash Q \vee R$
6. $(P \& Q) \rightarrow R, R \rightarrow \sim Q \vdash \sim P \vee \sim Q$
7. $\sim (P \& Q), \sim (\sim P \& Q) \vdash \sim Q$
8. $(P \rightarrow Q) \rightarrow P, \sim R \rightarrow \sim P \vdash R$
9. $(P \rightarrow Q) \rightarrow Q \vdash (Q \rightarrow P) \rightarrow P$
10. $(P \rightarrow Q) \vee (R \rightarrow S) \vdash (P \rightarrow S) \vee (R \rightarrow Q)$

II. Consider these alleged proofs. If they are correct, say so. If they are not correct, indicate where they are incorrect and why. (Note that poor strategy is not a mistake.)

1. $P \rightarrow (Q \rightarrow S) \vdash P \rightarrow (\sim Q \rightarrow \sim S)$

1	(1) $P \rightarrow (Q \rightarrow S)$	A
2	(2) P	A
3	(3) $\sim Q$	A
1,2	(4) $Q \rightarrow S$	1,2 $\rightarrow E$
1,2,3	(5) $\sim S$	3,4 MT
1,2	(6) $\sim Q \rightarrow \sim S$	5 $\rightarrow I$ (3)
1	(7) $P \rightarrow (\sim Q \rightarrow \sim S)$	6 $\rightarrow I$ (2)

2. $P \rightarrow R \vdash (Q \vee P) \rightarrow (S \vee R)$

1	(1) $P \rightarrow R$	A
2	(2) P	A
2	(3) $Q \vee P$	2 $\vee I$
1,2	(4) R	1,2 $\rightarrow E$
1,2	(5) $S \vee R$	4 $\vee I$
1	(6) $(Q \vee P) \rightarrow (S \vee R)$	5 $\rightarrow I$ (3)

3. $(P \rightarrow Q) \rightarrow P \vdash P$

1	(1) $(P \rightarrow Q) \rightarrow P$	A
2	(2) $\sim P$	A
3	(3) P	A
4	(4) $\sim Q$	A
2,3	(5) Q	2,3 RAA (4)
2	(6) $P \rightarrow Q$	5 $\rightarrow I$ (3)
1,2	(7) P	1,6 $\rightarrow E$
1	(8) P	2,7 RAA (2)

4. $P \rightarrow Q, \sim (P \& Q) \vee T, (\sim R \vee T) \vee \sim Q \vdash \sim P$

1	(1) $P \rightarrow Q$	A
2	(2) $\sim (P \& Q) \vee T$	A
3	(3) $(\sim R \vee T) \vee \sim Q$	A
4	(4) P	A
1,4	(5) Q	1,4 $\rightarrow E$
1,3,4	(6) $\sim R \vee T$	3,5 $\vee E$
1,4	(7) $P \& Q$	4,5 $\& I$
1,2,4	(8) T	2,7 $\vee E$
1,2,4	(9) $R \vee T$	8 $\vee I$
1,2,3	(10) $\sim P$	6,9 RAA(4)

TURN OVER!!!

III. There are three defendants – A, B, and C – and the following facts are known:

1. If A is innocent, then both B and C are guilty.
2. If B is guilty, then either A is innocent or C is innocent.
3. If B is innocent, then neither A nor C is guilty.
4. If C is guilty, then B is innocent.

Note that you do not know how many of these defendants are guilty. It may be 0, 1, 2, or all 3.

Who is innocent and who is guilty? Explain your reasoning. It is important to not only give the correct answer, or to show that the sentences would be true if your answer is correct, but to explain why your answer has to be the correct answer (i.e. is the only correct answer.)